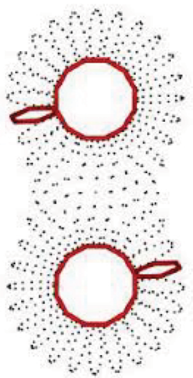
TEXAS STATE UNIVERSITY

The rising STAR of Texas



Texas State Topology Seminar

Tuesday, 2017, September 5, 12:30-1:50 p.m., in ENC (HPB) 143

Speaker: David Snyder

Topic: *Alexander-Spanier cohomology and Čech homology*

ABSTRACT

Alexander Duality promises that the cohomology of a compact subset X of Euclidean n-space E is isomorphic to the homology of E–X; for this to be true, one has to use Čech cohomology of X (it's not hard to find an example where it fails for singular cohomology). It is a well-known theorem of homotopy theory that the homotopy classes of maps of a "nice" space X (e.g. manifold or cellular complex) into the Eilenberg-MacLane $K(\pi, n)$ are in natural 1-1 correspondence with the elements of the cohomology group $H^1(X, \pi)$. If we wish to generalize this theorem to more general spaces X, it is necessary to use the Čech-type cohomology theory; singular cohomology will not do. In short, Čech homology is a necessity, despite its theoretical shortcomings. In this talk, we'll outline the Čech-Alexander-Spanier homology theory, using a exposition proposed by Massey that is based on a theorem by Nöbeling that avoids using the Continuum Hypothesis (but does use transfinite induction and hence is non-constructive). In particular, the relation between the classical Čech homology, based on nerves of coverings, and the Alexander-Spanier homology are related by a short exact sequence.