Texas State Topology Seminar

Friday, April 14, 11:00-noon, in DERR 229.

David Snyder
"PL Collapsibility And Simplicial Collapsibility of Complexes"
ABSTRACT

Collapsibility is a notion introduced by Whitehead in 1939. All collapsible simplicial complexes are contractible as topological spaces, but the converse is false (Bing's House is one example). Collapsibility is central to PL (piecewise linear) topology, particularly in studying triangulations of manifolds. Zeeman's Dunce Hat is a contractible 2-dimensional space that is contractible but does *not* admit a collapsible triangulation. Characterizing those complexes that are collapsible is hard. There are only a few criteria available:

 (a) all cones are collapsible;
 (b) (Chillingsworth's criterion [1967]) all subdivisions of convex 3-polytopes are collapsible; and
 (c) (Crowley's criterion [2007]) all 3-d pseudomanifolds that are CAT(0) under the
     "equilateral flat metric" are collapsible.

In this talk, we will survey selected results from the history of the study of collapsibility, we compare piecewise linear collapsing to combinatorial collapsing, and we discuss in that context a recently-published paper of Adiprasito and Benedetti which provides a combinatorial version of a result by Ancel and Guilbault (geometric topologists from the R.L. Moore/R.H. Bing "Texas Topology" tradition) regarding contractibility of CAT(0) spaces. In particular, in that paper there is an interesting consequence regarding triangulations of certain cell-like manifolds-with-boundary in higher dimensions. The paper also extends some of the characterizations above to higher-dimensional complexes. We'll end the talk with some open questions, including one posed by Gromov.